Steady State Cornering

Timothy T. Maxwell
Advanced Vehicle Engineering Laboratory
Texas Tech University

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Handling — responsiveness of vehicle to driver inputs or ease of control

Handling is measure of the driver–vehicle closed–loop system

Vehicle only must be characterized as open–loop system

Vehicle response to steering input or directional response

Most common measure of open–loop response is the Understeer Gradient

Understeer Gradient only valid for steady state
At low speed (parking lot maneuvers) tires need not develop lateral forces.

- Tires roll with no slip angle — center of turn must lie off projection of rear axle.
- Perpendiculars from front wheels pass through same turn center.
Low Speed Turning

- Ideal turning angles
  \[ \delta_o = \frac{L}{R + t/2} \]
  \[ \delta_i = \frac{L}{R - t/2} \]

- Average angle is the Ackerman Angle
  \[ \delta \approx \frac{L}{R} \]
Steady State Cornering

Ackerman Angle (Low Speed)

- **Ackerman angle or Ackerman Geometry**
  - Exact geometry of front wheels as on previous slide
  - Correct angles depend on vehicle wheelbase & turn angle
  - Deviations from Ackerman angles for the right or left steer angles can significantly affect tire wear
  - Deviations do not significantly affect directional response for low speed turns
  - Deviations do affect steering torques
  - With correct Ackerman geometry, steering torques increase with steer angle and provide feedback to driver
  - With parallel steering, steering torque increases initially and then decreases
    - Steering torque can become negative so that vehicle turns more deeply into turn
Off–Tracking at Low Speed

- Off–tracking at rear wheels
  - Off–tracking distance
    \[ \Delta = R \left[ 1 - \cos \left( \frac{L}{R} \right) \right] \]
  - Recall series form of \( \cos \)
    \[ \cos z = 1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \frac{z^6}{6!} + \frac{z^8}{8!} \ldots \]
  - Then
    \[ \Delta = \frac{L^2}{2R} \]

- Off–tracking is primarily a concern for long wheelbase vehicles like trucks and buses
- For articulated vehicle — Tractrix equations
High Speed Cornering

- High speed cornering produces different equations wrt low speed cornering
- Tires must develop significant lateral forces to counteract the lateral acceleration
- *Slip angles* will be present at each wheel
Tire Cornering Forces

- Tire slip–angle between direction of heading and direction of travel
- Lateral or cornering force grows with slip angle
**Slip Angles**

- **Below about 5° slip relationship is linear**

  \[ F_y = C_\alpha \alpha \]

- **\( C_\alpha \) — cornering stiffness**

- **Positive slip angle produces negative force (to the left) on tire**

- **Thus, \( C_\alpha \) must be negative**

- **SAE defines \( C_\alpha \) as the negative of the slope**
Steady State Cornering

Slip Angle

- Cornering stiffness depends on several variables
  - Tire size
  - Tire type (radial or bias ply)
  - Wheel width
  - Tread design
  - Number of plies
  - Cord angles
  - Load
  - Inflation pressure

- Speed not a strong influence on cornering forces produced by tire
**Cornering Coefficient**

- Cornering stiffness divided by load

\[ CC_\alpha = \frac{C_\alpha}{F_y} \left[ \frac{\text{lbf}_y}{\text{lbf}_z} / \text{deg} \right] \]

- Cornering force — strong dependence on load

- Cornering coefficient largest at light load and diminishes as load reaches rated value

- Tire & Rim Association rated load

- At 100% load — cornering coefficient approximately

0.2 lbf cornering force / lbf load / deg slip angle
Slip Angle Dependence

Steady State Cornering
Cornering Equations

- Apply NSL and description of geometry in turns
  - Bicycle model
  - At high speed turn radius >> wheelbase
    assume small angles – inner and outer slip angles same
  - Both front wheels represented by one with steer angle $\delta$
  - Sum forces in lateral direction

$$\sum F_y = F_{yf} + F_{yr} = \frac{MV^2}{R}$$

$F_{yf}$ = cornering force at front
$F_{yr}$ = cornering force at rear
$M$ = mass of vehicle
$V$ = forward velocity
$R$ = turn radius
Cornering Equations

- For vehicle to be in equilibrium
  \[ F_{yf}b - F_{yr}c = 0 \]

- Slip angles are
  \[ \alpha_f = \frac{W_f V^2}{C_{\alpha_f} g R} \]
  \[ \alpha_r = \frac{W_r V^2}{C_{\alpha_r} g R} \]

- Geometry gives
  \[ \delta = 57.3 \frac{L}{R} + \alpha_f + \alpha_r \]

- Combining
  \[ \delta = 57.3 \frac{L}{R} + \left( \frac{W_f}{C_{\alpha_f}} - \frac{W_r}{C_{\alpha_r}} \right) \left( \frac{V^2}{g R} \right) \]
Understeer Gradient

- Previous equation is the *Understeer Gradient*
  
  \[ \delta = 57.3 \frac{L}{R} + ka_y \]

- Lateral acceleration
  
  \[ \frac{V^2}{gR} \]

- Magnitude and direction of steering inputs required
  
  \[ \frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}} \]
Understeer Gradient

- **Neutral Steer**
  \[
  \frac{W_f}{C_{\alpha_f}} = \frac{W_r}{C_{\alpha_r}} \implies K = 0 \implies \alpha_f = \alpha_r
  \]

- **Understeer**
  \[
  \frac{W_f}{C_{\alpha_f}} > \frac{W_r}{C_{\alpha_r}} \implies K > 0 \implies \alpha_f > \alpha_r
  \]

- **Oversteer**
  \[
  \frac{W_f}{C_{\alpha_f}} < \frac{W_r}{C_{\alpha_r}} \implies K < 0 \implies \alpha_f < \alpha_r
  \]